OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4730

Mechanics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Monday 19 January 2009
Afternoon

Duration: 1 hour 30 minutes

4730

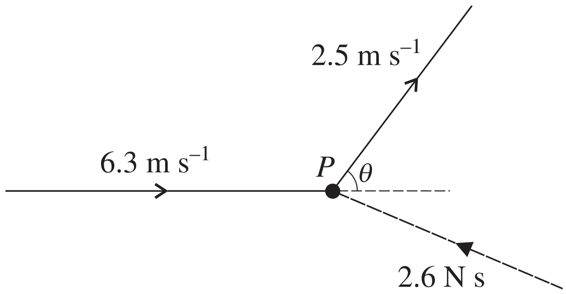
INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1



A particle P of mass 0.5 kg is moving in a straight line with speed 6.3 m s^{-1}. An impulse of magnitude 2.6 N s applied to P deflects its direction of motion through an angle θ, and reduces its speed to 2.5 m s^{-1} (see diagram). By considering an impulse-momentum triangle, or otherwise,

(i) show that $\cos\theta = 0.6$, **[4]**

(ii) find the angle that the impulse makes with the original direction of motion of P. **[4]**

2

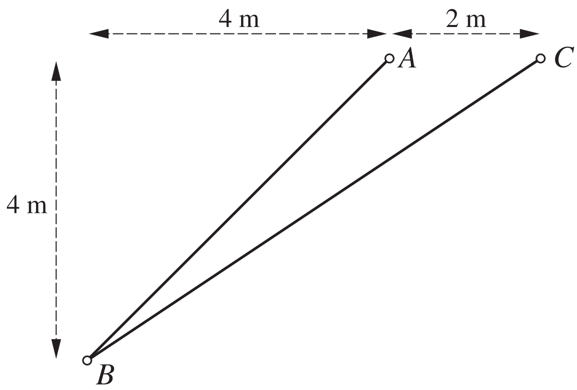


Fig. 1

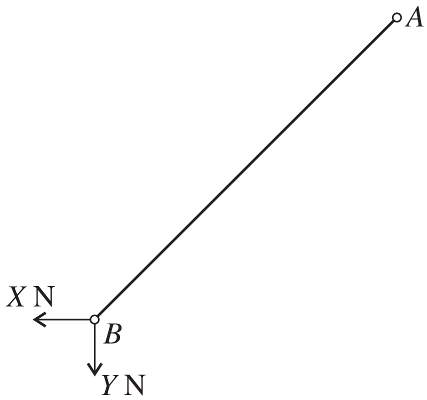


Fig. 2

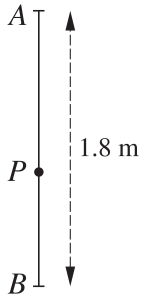
Two uniform rods AB and BC, of weights 70 N and 110 N respectively, are freely jointed at B. The rods are in equilibrium in a vertical plane with A and C at the same horizontal level and AC = 2 m. The rod AB is freely jointed to a fixed point at A and the rod BC is freely jointed to a fixed point at C. The horizontal distance between B and A is 4 m and B is 4 m below AC; angle BAC is obtuse (see Fig. 1). The force exerted on the rod AB at B, by the rod BC, has horizontal and vertical components as shown in Fig. 2.

(i) By taking moments about A for the rod AB find the value of $X - Y$. **[2]**

(ii) By taking moments about C for the rod BC show that $2X - 3Y + 165 = 0$. **[2]**

(iii) Find the magnitude of the force acting between AB and BC at B. **[4]**

3



A and B are fixed points with B at a distance of 1.8 m vertically below A. One end of a light elastic string of natural length 0.6 m and modulus of elasticity 24 N is attached to A, and one end of an identical elastic string is attached to B. A particle P of weight 12 N is attached to the other ends of the strings (see diagram).

(i) Verify that P is in equilibrium when it is at a distance of 1.05 m vertically below A. **[2]**

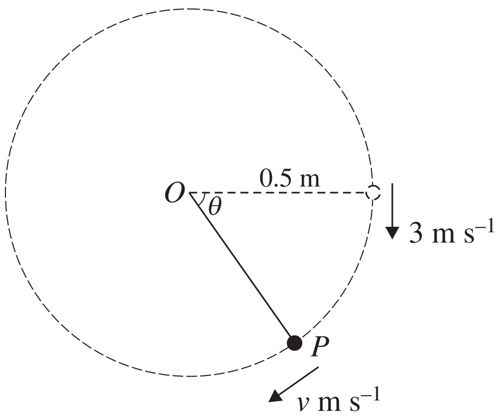
P is released from rest at the point 1.2 m vertically below A and begins to move.

(ii) Show that, when P is x m below its equilibrium position, the tensions in PA and PB are $(18 + 40x)$ N and $(6 - 40x)$ N respectively. **[2]**

(iii) Show that P moves with simple harmonic motion of period 0.777 s, correct to 3 significant figures. **[3]**

(iv) Find the speed with which P passes through the equilibrium position. **[2]**

4



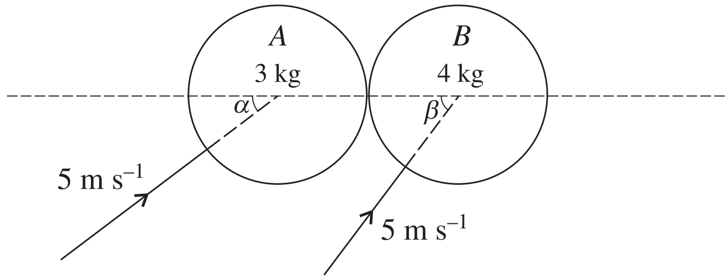
One end of a light inextensible string of length 0.5 m is attached to a fixed point O. A particle P of mass 0.2 kg is attached to the other end of the string. With the string taut and horizontal, P is projected with a velocity of $3\,\text{m}\,\text{s}^{-1}$ vertically downward. P begins to move in a vertical circle with centre O. While the string remains taut the angular displacement of OP is θ radians from its initial position, and the speed of P is $v\,\text{m}\,\text{s}^{-1}$ (see diagram).

(i) Show that $v^2 = 9 + 9.8 \sin \theta$. **[3]**

(ii) Find, in terms of θ, the radial and tangential components of the acceleration of P. **[3]**

(iii) Show that the tension in the string is $(3.6 + 5.88 \sin \theta)$ N and hence find the value of θ at the instant when the string becomes slack, giving your answer correct to 1 decimal place. **[4]**

5



Two smooth uniform spheres A and B, of equal radius, have masses 3 kg and 4 kg respectively. They are moving on a horizontal surface, each with speed $5\,\text{m}\,\text{s}^{-1}$, when they collide. The directions of motion of A and B make angles α and β respectively with the line of centres of the spheres, where $\sin\alpha = \cos\beta = 0.6$ (see diagram). The coefficient of restitution between the spheres is 0.75. Find the angle that the velocity of A makes, immediately after impact, with the line of centres of the spheres. **[10]**

6 A stone of mass 0.125 kg falls freely under gravity, from rest, until it has travelled a distance of 10 m. The stone then continues to fall in a medium which exerts an upward resisting force of $0.025v$ N, where $v\,\text{m}\,\text{s}^{-1}$ is the speed of the stone t s after the instant that it enters the resisting medium.

(i) Show by integration that $v = 49 - 35e^{-0.2t}$. **[8]**

(ii) Find how far the stone travels during the first 3 seconds in the medium. **[4]**

7 A particle of mass 0.8 kg is attached to one end of a light elastic string of natural length 2 m and modulus of elasticity 20 N. The other end of the string is attached to a fixed point O. The particle is held at rest at O and then released. When the extension of the string is x m, the particle is moving with speed $v\,\text{m}\,\text{s}^{-1}$.

(i) By considering energy show that $v^2 = 39.2 + 19.6x - 12.5x^2$. **[4]**

(ii) Hence find

(a) the maximum extension of the string, **[2]**

(b) the maximum speed of the particle, **[4]**

(c) the maximum magnitude of the acceleration of the particle. **[5]**

4730 Mechanics 3

1 (i)	For triangle sketched with sides (0.5)2.5 and (0.5)6.3 and angle θ correctly marked OR Changes of velocity in i and j directions $2.5\cos\theta - 6.3$ and $2.5\sin\theta$, respectively.	B1	May be implied in subsequent working.
	For sides 0.5x2.5, 0.5x6.3 and 2.6 (or 2.5, 6.3 and 5.2) OR $-2.6\cos\alpha = 0.5(2.5\cos\theta - 6.3)$ and $2.6\sin\alpha = 0.5(2.5\sin\theta)$	B1ft	May be implied in subsequent working.
	$[5.2^2 = 2.5^2 + 6.3^2 - 2\text{x}2.5\text{x}6.3\cos\theta$ OR $2.6^2 = 0.5^2\{(2.5\cos\theta - 6.3)^2 + (2.5\sin\theta)^2\}]$	M1	For using cosine rule in triangle or eliminating α.
	$\cos\theta = 0.6$	A1 [4]	AG
(ii)		M1	For appropriate use of the sine rule or substituting for θ in one of the above equations in θ and α
	$\sin\alpha = 2.5\text{x}0.8/5.2$ OR $-2.6\cos\alpha = 0.5(2.5\text{x}0.6 - 6.3)$	A1	
		M1	For evaluating $(180 - \alpha)^o$ or $(\pi - \alpha)^c$
	Impulse makes angle of 157^o or 2.75^c with original direction of motion of P.	A1 [4]	SR (relating to previous 2 marks; max 1 mark out of 2) $\alpha = 23^o$ or 0.395^C B1

2 (i)	[70x2 = 4X – 4Y]	M1	For taking moments about A for AB (3 terms needed)
	X – Y = 35	A1 [2]	
(ii)	[110x3 = -4X + 6Y]	M1	For taking moments about C for BC (3 terms needed)
	2X – 3Y + 165 = 0	A1 [2]	AG
(iii)		M1	For attempting to solve for X and Y
	X = 270, Y = 235	A1ft	ft any (X, Y) satisfying the equation given in (ii)
		M1	For using magnitude = $\sqrt{X^2 + Y^2}$
	Magnitude is 358N	A1ft [4]	ft depends on all 4 Ms

.

3 (i)	[T_A = (24x0.45)/0.6, T_B = (24x0.15)/0.6] $T_A - T_B$ = 18 – 6 = 12 = W ➔ P in equil'm.	M1 A1 [2]	For using T = λ x/L for PA or PB
(ii)	Extensions are 0.45 + x and 0.15 – x Tensions are 18 + 40x and 6 – 40x	B1 B1 [2]	AG From T = λ x/L for PA and PB
(iii)	[12 + (6 – 40x) – (18 + 40x) = 12$\ddot{x}$/g] $\ddot{x}$ = -80gx/12 ➔ SHM Period is 0.777s	M1 A1 A1 [3]	For using Newton's second law (4 terms required) AG From Period = $2\pi\sqrt{12/(80g)}$
(iv)	[v_{max} = $0.15\sqrt{80g/12}$ or v_{max} = 2πx0.15/0.777 or ½ (12/g)v_{max}^2 +mg(0.15) +24{0.45^2 + 0.15^2 -0.6^2}/(2x0.6) = 0] Speed is 1.21ms^{-1}	M1 A1 [2]	For using v_{max} = An or v_{max} = 2πA/T or conservation of energy (5 terms needed)

4 (i)	Loss in PE = mg(0.5sinθ) [½ mv^2 – ½ m3^2 = mg(0.5sinθ)] v^2 = 9 + 9.8sinθ	B1 M1 A1 [3]	For using KE gain = PE loss (3 terms required) AG
(ii)	a_r = 18 + 19.6sinθ [ma_t = mg cosθ] a_t = 9.8cosθ	B1 M1 A1 [3]	Using a_r = v^2/0.5 For using Newton's second law tangentially
(iii)	[T - mg sinθ = ma_r] T – 1.96sinθ = 0.2(18 + 19.6sinθ) T = 3.6 + 5.88sinθ θ = 3.8	M1 A1 A1 B1 [4]	For using Newton's second law radially (3 terms required) AG

5	Initial **i** components of velocity for A and B are $4ms^{-1}$ and $3ms^{-1}$ respectively. 3x4 + 4x3 = 3a + 4b 0.75(4 – 3) = b – a a = 3 Final **j** component of velocity for A is $3ms^{-1}$ Angle with l.o.c. is 45^o or 135^O	B1 M1 A1 M1 A1 M1 A1 B1 M1 A1ft [10]	May be implied. For using p.c.mmtm. parallel to l.o.c. For using NEL For attempting to find a Depends on all three M marks May be implied For using $\tan^{-1}(v_j/v_i)$ for A ft incorrect value of a ($\neq 0$) only
			SR for consistent sin/cos mix (max 8/10) 3x3 + 4x4 = 3a + 4b and b – a = 0.75(3 – 4) M1 M1 as scheme and A1 for *both* equ's a = 4 M1 as scheme A1 j component for A is $4ms^{-1}$ B1 Angle $\tan^{-1}(4/4) = 45^o$ M1 as scheme A1

6(i)	Initial speed in medium is $\sqrt{2g \times 10}$ (= 14) [0.125dv/dt = 0.125g – 0.025v] $\int \frac{5dv}{5g - v} = \int dt$ -5 ln(5g – v) = t (+A) [-5 ln35 = A] t = 5 ln{35/(49 – v)} $v = 49 - 35e^{-0.2t}$	B1 M1 M1 A1 M1 A1 M1 A1 [8]	 For using Newton's second law with a = dv/dt (3 terms required) For separating variables and attempt to integrate For using v(0) = 14 For method of transposition AG
(ii)	 $x = 49t + 175e^{-0.2t}$ (+B) $[x(3) = (49 \times 3 + 175e^{-0.6}) - (0 + 175)]$ Distance is 68.0m	M1 A1 M1 A1 [4]	For integrating to find x(t) For using limits 0 to 3 or for using x(0) = 0 and evaluating x(3)

7(i)	Gain in EE = $20x^2/(2 \times 2)$ Loss in GPE = $0.8g(2 + x)$ [$\frac{1}{2}0.8v^2 = (15.68 + 7.84x) - 5x^2$] $v^2 = 39.2 + 19.6x - 12.5x^2$	B1 B1 M1 A1 [4]	Accept 0.8gx if gain in KE is $\frac{1}{2}0.8(v^2 - 19.6)$ For using the p.c.energy AG
(ii)	**(a)** Maximum extension is 2.72m	M1 A1 [2]	For attempting to solve $v^2 = 0$
	(b) [$19.6 - 25x = 0$, $v^2 = 46.8832 - 12.5(x - 0.784)^2$] x = 0.784 or c = 46.9 [$v_{max}^2 = 39.2 + 15.3664 - 7.6832$] Maximum speed is 6.85ms^{-1}	 M1 A1 M1 A1 [4]	For solving 20x/2 = 0.8g or for differentiating and attempting to solve $d(v^2)/dx = 0$ or $dv/dx = 0$ or for expressing v^2 in the form $c - a(x - b)^2$. For substituting x = 0.784 in the expression for v^2 or for evaluating $\sqrt{c}$
	(c) $\pm(0.8g - 20x/2) = 0.8a$ or $2v\, dv/dx = 19.6 - 25x$ $a = \pm(9.8 - 12.5x)$ or $\ddot{y} = -12.5y$ where $y = x - 0.784$ [$\|a\|_{max} = \|9.8 - 12.5 \times 2.72\|$ or $\|\ddot{y}_{max}\| = \|-12.5(2.72 - 0.784)\|$] Maximum magnitude is 24.2ms^{-2}	 M1 A1 A1 M1 A1 [5]	For using Newton's second law (3 terms required) or a = v dv/dx For substituting x = ans(ii)(a) into a(x) or y = ans(ii)(a) – 0.784 into $\ddot{y}$ (y)